Financial [114 marks]

1. [Maximum mark: 6]

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5 %, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four significant figures. [3]

Markscheme

$\left(1+\frac{5.5}{4\times100}\right)^4$ ***(M1)(A1)***

1.056 ***A1***

[3 marks]

(b) Laurie makes no further deposits to or withdrawals from the account.

Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

Markscheme

EITHER

$2P=P\times\left(1+\frac{5.5}{100\times4}\right)^{4n}$ **OR** $2P=P\times(\text{their }(a))^m$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

OR

PV = ±1
FV = ∓1
I% = 5.5
P/Y = 4
C/Y = 4
n = 50.756… ***(M1)(A1)***

OR

PV = ±1
FV = ∓2
I% = 100(their (a) − 1)
P/Y = 1
C/Y = 1 ***(M1)(A1)***

THEN

⇒ 12.7 years

Laurie will have double the amount she invested during 2032 ***A1***

[3 marks]

2. [Maximum mark: 12]

Helen and Jane both commence new jobs each starting on an annual salary of $\$70,000$. At the start of each new year, Helen receives an annual salary increase of $\$2400$.

Let $\$H_n$ represent Helen's annual salary at the start of her nth year of employment.

(a) Show that $H_n = 2400n + 67\,600$. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

uses $H_n = H_1 + (n-1)d$ with $H_1 = 70\,000$ and $d = 2400$ **(M1)**

$H_n = 70\,000 + 2400(n-1)$ **A1**

so $H_n = 2400n + 67\,600$ **AG**

[2 marks]

At the start of each new year, Jane receives an annual salary increase of 3% of her previous year's annual salary.

Jane's annual salary, $\$J_n$, at the start of her nth year of employment is given by $J_n = 70\,000(1.03)^{n-1}$.

(b) Given that J_n follows a geometric sequence, state the value of the common ratio, r. [1]

Markscheme

$r = 1.03$ **A1**

[1 mark]

At the start of year N, Jane's annual salary exceeds Helen's annual salary for the first time.

(c.i) Find the value of N. [3]

Markscheme

evidence of use of an appropriate table or graph or GDC numerical solve feature to find the value of N such that $J_n > H_n$ **(M1)**

EITHER

for example, an excerpt from an appropriate table

N	H_n	J_n
11	94 000	94 074

(A1)

OR

for example, use of a GDC numerical solve feature to obtain $N = 10.800\ldots$ **(A1)**

Note: Award **A1** for an appropriate graph. Condone use of a continuous graph.

THEN

$N = 11$ **A1**

[3 marks]

(c.ii) For the value of N found in part (c) (i), state Helen's annual salary and Jane's annual salary, correct to the nearest dollar. [2]

Markscheme

$H_{11} = 94000\,(\$)$ **A1**

$J_{11} = 94074\,(\$)$ **A1**

Helen's annual salary is $\$94000$ and Jane's annual salary is $\$94074$

Note: Award **A1** for a correct H_{11} value and **A1** for a correct J_{11} value seen in part (c) (i).

[2 marks]

(d) Find Jane's total earnings at the start of her 10th year of employment. Give your answer correct to the nearest dollar. [4]

Markscheme

at the start of the 10th year, Jane will have worked for 9 years so the value of S_9 is required **R1**

Note: Award **R1** if S_9 is seen anywhere.

uses $S_n = \frac{J_1(r^n-1)}{r-1}$ with $J_1 = 70\,000, r = 1.03$ and $n = 9$ **(M1)**

Note: Award **M1** if $n = 10$ is used.

$S_9 = \frac{70\,000\left((1.03)^9-1\right)}{1.03-1} = 711\,137.42\ldots$ **(A1)**

$= 711\,137\,(\$)$

Jane's total earnings are $\$711\,137$ (correct to the nearest dollar)

[4 marks]

3. [Maximum mark: 12]

Andy and Jess each have $\$5000$.

Andy invests the money in a new savings plan that will pay interest at the end of each month.

Andy will receive a fixed amount of interest each month. The amount received is 0.315% of the initial investment.

(a.i) Determine the amount of interest Andy will receive at the end of each month. Give this answer correct to two decimal places. [1]

Markscheme

15.75 ***A1***

[1 mark]

(a.ii) Hence, determine the amount of interest Andy will receive each year. [1]

Markscheme

189 ***A1***

[1 mark]

(a.iii) Write down an expression in the form $5000 + pn$, where $p \in \mathbb{Z}^+$, for the amount of money Andy will have in his savings plan at the end of n years.

[1]

Markscheme

$5000 + 189n$ ***A1***

[1 mark]

(a.iv) Hence, show that Andy will have $\$5945$ in his savings plan at the end of 5 years.

[1]

Markscheme

$5000 + 189 \times 5$ ***A1***

$= 5945$ ***AG***

[1 mark]

In this part, where appropriate, give all answers to the nearest dollar.

Jess invests her $\$5000$ in a new account that pays 3% interest compounded annually.

(b.i) Determine the amount of money that will be in Jess's account at the end of 5 years. [2]

Markscheme

EITHER

attempt to use formula $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ with $n = 5$ ***(M1)***

$FV = 5000\left(1 + \frac{3}{100}\right)^5$ **OR** $5000(1 + 0.03)^5$

OR

attempt to use GDC app (at least two correct entries seen) ***(M1)***

$PV = 5000$, $I = 3\%, n = 5, P/Y = C/Y = 1$

OR

$5150, 5304.5, 5463.635, 5627.54\ldots, 5796.37\ldots$ ***(M1)***

THEN

$5796.37\ldots$

5796 ***A1***

[2 marks]

(b.ii) Hence, find the amount of interest Jess will receive in the 5 years. [1]

Markscheme

$796 (= 796.370\ldots)$ ***A1***

[1 mark]

(c.i) Write an expression in the form $5000 \times q^n$, where $q \in \mathbb{R}^+$, for the amount of money that Jess will have in her account at the end of n years. [2]

Markscheme

$FV = PV + \left(1 + \frac{r}{100k}\right)^{kn}$ or equivalent with $K = 1$ ***(M1)***

$5000(1 + 0.03)^n$

$5000(1.03)^n$ ***A1***

[2 marks]

(c.ii) Hence, determine the smallest number of complete years that it will take for Jess to have more money in her account than Andy has in his savings plan. [3]

Markscheme

setting $5000(1.03)^n \geq 5000 + 189n$ ***(M1)***

Note: Condone equality but do not accept $5000(1.03)^n \leq 5000 + 189n$.

attempt to solve their inequality or equation to find n ***(M1)***

$16.0097\ldots$ seen **OR** either $8023.53\ldots < 8024$ or $8264.23\ldots > 8213$

17 (years) ***A1***

[3 marks]

4. [Maximum mark: 7]

Bob invests $1\,000$ dinar in an account which pays a nominal annual interest rate of 4% compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1\,000(1+k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of k. [1]

Markscheme

$k = \frac{4}{400} \left(= \frac{1}{100} = 0.01\right)$ ***A1***

[1 mark]

(b) Expand and simplify $(1+x)^4$. [2]

Markscheme

attempt to find binomial coefficients or multiply out brackets ***(M1)***

e.g. Pascal's triangle down to correct row **OR** $\left(1+2x+x^2\right)^2$ **OR** substitute into binomial expansion

$(1+x)^4$

$= 1 + 4x + 6x^2 + 4x^3 + x^4$ ***A1***

[2 marks]

(c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

Markscheme

METHOD 1

recognition that the expansion can be used with x replaced with k ***(M1)***

$\left(1+\frac{1}{100}\right)^4$

$=1+\frac{4}{100}+\frac{6}{100^2}+\ldots(=1+0.04+0.0006+\ldots)$ ***(A1)***

multiplies by 1000 (seen anywhere) ***(M1)***

$1000\left(1+\frac{1}{100}\right)^4$

$1000+40+0.6+\ldots(=1040.6\ldots)$

$=1041$ (dinar) ***A1***

METHOD 2

attempt to find the value of $(1+k)^4$ by hand ***(M1)***

$(1.01)^4=(1.0201)(1.01)^2=(1.030301)(1.01)$

$=1.0406\ldots$ ***(A1)***

multiplies by 1000 (seen anywhere) ***(M1)***

$1000(1.01)^4$

$=1040.6\ldots$

$=1041$ (dinar) ***A1***

[4 marks]

5. [Maximum mark: 7]

Darren buys a car for $\$35\,000$. The value of the car decreases by $15\,\%$ in the first year.

(a) Find the value of the car at the end of the first year. [2]

Markscheme

recognition that a $15\,\%$ loss leaves $85\,\%$ OR finding $15\,\%$ and subtracting from original ***(M1)***

$0.85 \times 35\,000$ OR $35\,000 - 0.15 \times 35\,000$

$= (\$)29\,750$ ***A1***

Note: Accept $(\$)29\,800$.

[2 marks]

After the first year, the value of the car decreases by $11\,\%$ in each subsequent year.

(b) Find the value of Darren's car 10 years after he buys it, giving your answer to the nearest dollar. [2]

Markscheme

EITHER

$29\,750 \times 0.89^9$ ***(A1)***

OR

$\mathrm{N} = 9$

$\mathrm{I\%} = -11$

$\mathrm{PV} = \mp 29\,750$ ***(A1)***

THEN

value $(FV) = (\$)10\,423$ ***A1***

Note: For this ***A1*** the answer must be rounded to the nearest dollar. Accept $(\$)10\,441$ from using 3 sf answer from part (a).

[2 marks]

When Darren has owned the car for n complete years, the value of the car is less than $10\,\%$ of its original value.

(c) Find the least value of n. [3]

Markscheme

METHOD 1

attempt to solve the inequality (or equation) $29\,750 \times 0.89^{n-1} < 3500$ OR table of values *(M1)*

$19.3643\ldots$ OR $(n = 19 \Rightarrow) 3651.80\ldots$ OR $(n = 20 \Rightarrow) 3250.10\ldots$ *(A1)*

Note: For candidates using
$(\$)\,29\,800,\ n > 19.3787\ldots,\ (n = 19 \Rightarrow) 3657.93\ldots,\ (n = 20 \Rightarrow) 3255.56\ldots$.

$n = 20$ *A1*

METHOD 2

use of the finance app with $\text{I}\% = -11,\ \text{PV} = \mp 29\,750,\ \text{FV} = \pm 3500$

OR $29\,750 \times 0.89^{N} < 3500$ (condone the use of n or x) *(M1)*

$(N =) 18.3643\ldots$ *(A1)*

Note: For candidates using $(\$)\,29\,800,\ N = 18.3787\ldots$

$n = 20$ *A1*

[3 marks]

6. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is $\$4200$.

Option B The first payment is $\$1500$. In each month which follows, the payment is 4% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(a.i) Option A; [2]

Markscheme

4200×36 ***(A1)***

$= 151200$

$= (\$)151000$ ***A1***

[2 marks]

(a.ii) Option B. [3]

Markscheme

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

recognizing sum of a geometric sequence is required ***(M1)***

$\frac{1500(1-1.04^{36})}{1-1.04}$ ***(A1)***

$= 116397.4707\ldots$

$= (\$)116397.47$ ***A1***

[3 marks]

Sorin received an inheritance of $\$160\,000$. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 5% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years. [1]

Markscheme

Sorin's future value after n years $= 160000\left(1+\frac{5}{100\times12}\right)^{12n}$ ***A1***

[1 mark]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c.i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month. [3]

Markscheme

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

Sorin's total $160000\left(1+\frac{5}{100\times12}\right)^6\ (164041.89\ldots)$ ***(A1)***

Daniela's total $=\frac{1500\left(1-1.04^6\right)}{1-1.04}(=9949.46\cdots)$ ***(A1)***

total value $=(\$)173991.36$ ***A1***

[3 marks]

(c.ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least $\$257\,000$. [3]

Markscheme

EITHER (finding number of months, m)

$160000\left(1+\frac{5}{100\times12}\right)^m+\frac{1500(1-1.04^m)}{1-1.04}\ (\geq 257000)$ ***(A1)***

$m\geq 28.4412\ldots$ OR $(m=28\Rightarrow)\ 254707$ AND $(m=29\Rightarrow)\ 259954$ ***(A1)***

Note: Condone use of an equation or strict inequality.

OR (finding number of years, n)

$160000\left(1+\frac{5}{100\times12}\right)^{12\times n}+\frac{1500\left(1-1.04^{12\times n}\right)}{1-1.04}\ (\geq 257000)$ ***(A1)***

$n\geq 2.37010\ldots$(years) ***(A1)***

Note: Condone use of an equation or strict inequality.

THEN

$m=29$ (months) ***A1***

[3 marks]

At the end of the three years, Daniela invested $\$30\,000$ for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

(d) Find the value of r if this investment grows to $\$41\,000$ after six years. [3]

Markscheme

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

EITHER

$\mathrm{N} = 24$

$\mathrm{PV} = \mp 30000$

$\mathrm{PMT} = 0$

$\mathrm{FV} = \mp 41000$

$\mathrm{P/Y} = 4$

$\mathrm{C/Y} = 4$

OR

$\mathrm{N} = 6$

$\mathrm{PV} = \mp 30000$

$\mathrm{PMT} = 0$

$\mathrm{FV} = \pm 41000$

$\mathrm{P/Y} = 1$

$\mathrm{C/Y} = 4$

(M1)(A1)

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***(A1)*** for all entries correct. PV and FV must have opposite signs**.**

OR

$30000\left(1+\frac{r}{100\times 4}\right)^{6\times 4}=41000$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempting to substitute into compound interest formula, award ***(A1)*** for correct equation.

THEN

$5.24027\ldots$

$(r=)5.24\%$ ***A1***

[3 marks]

7. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is $\$5500$.

Option B The first payment is $\$2000$. In each month which follows, the payment is 6% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(a.i) Option A; [2]

Markscheme

5500×36 ***(A1)***

$= (\$)198000$ ***A1***

[2 marks]

(a.ii) Option B. [3]

Markscheme

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** **i**n that part is not awarded.

recognizing sum of a geometric sequence is required ***(M1)***

$\frac{2000(1-1.06^{36})}{1-1.06}$ ***(A1)***

$= 238241.7333\ldots$

$= (\$)238241.73$ ***A1***

[3 marks]

Sorin received an inheritance of $\$120\,000$. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 4% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years. [1]

Markscheme

Sorin's future value after n years $= 120000\left(1 + \frac{4}{100 \times 12}\right)^{12n}$ ***A1***

[1 mark]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c.i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month. [3]

Markscheme

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

Sorin's total $= 120000\left(1 + \frac{4}{100 \times 12}\right)^{6} (= 122420.09)$ ***(A1)***

Daniela's total $= \frac{2000\left(1 - 1.06^{6}\right)}{1 - 1.06} (= 13950.64)$ ***(A1)***

total value $= (\$)136370.73$ ***A1***

[3 marks]

(c.ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least $\$250\,000$. [3]

Markscheme

EITHER (finding number of months, m)

$120000\left(1 + \frac{4}{100 \times 12}\right)^{m} + \frac{2000(1 - 1.06^{m})}{1 - 1.06} (\geq 250000)$ ***(A1)***

$m \geq 26.0905$ OR $(m = 26 \Rightarrow)\ 249157\ldots$ AND $(m = 27 \Rightarrow)\ 258692\ldots$ ***(A1)***

Note: Condone use of an equation or strict inequality.

OR (finding number of years, n)

$120000\left(1 + \frac{4}{100 \times 12}\right)^{12 \times n} + \frac{2000\left(1 - 1.06^{12 \times n}\right)}{1 - 1.06} (\geq 250000)$ ***(A1)***

$n \geq 2.17421\ldots$(years) ***(A1)***

Note: Condone use of an equation or strict inequality.

THEN

$m = 27(\text{months})$ ***A1***

[3 marks]

At the end of the three years, Daniela invested $\$40\,000$ for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

(d) Find the value of r if this investment grows to $\$53\,000$ after six years. [3]

Markscheme

Note: The first time an answer is not given to two decimal places in parts (a)(ii), (c)(i) or (d), the final ***A1*** in that part is not awarded.

EITHER

$\text{N} = 24$

$\text{PV} = \mp 40000$

$\text{PMT} = 0$

$\text{FV} = \pm 53000$

$\text{P/Y} = 4$

$\text{C/Y} = 4$

OR

$\text{N} = 6$

$\text{PV} = \mp 40000$

$\text{PMT} = 0$

$\text{FV} = \pm 530000$

$\text{P/Y} = 1$

$\text{C/Y} = 4$

(M1)(A1)

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. PV and FV must have opposite signs.

OR

$40000\left(1 + \frac{r}{100 \times 4}\right)^{6 \times 4} = 53000$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempting to substitute into compound interest formula, award ***(A1)*** for correct equation.

THEN

$4.71781\ldots$

$(r =)4.72\%$ ***A1***

[3 marks]

8. [Maximum mark: 6]

The value of a car is given by the function $C = 40\,000(0.91)^t$, where t is in years since 1 January 2023 and C is in USD ($).

(a) Write down the annual rate of depreciation of the car. [1]

Markscheme

9% (accept 0.09) ***A1***

[1 mark]

(b) Find the value of the car on 1 January 2028. [2]

Markscheme

$t = 5$ (seen anywhere) ***(A1)***

$24961.28\ldots$

25000 (dollars) ***A1***

[2 marks]

Alvie wants to buy this car. On 1 January 2023, he invested $\$15\,000$ in an account that earns 3% annual interest compounded yearly. He makes no further deposits to, or withdrawals from, the account.

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra $\$M$.

(c) Find the value of M. [3]

Markscheme

EITHER

$n = 5$

$\mathrm{I\%} = 3$

$\mathrm{PV} = (\pm)15000$

$\mathrm{P/Y} = 1$

$\mathrm{C/Y} = 1$ ***(A1)***

Note: Award ***(A1)*** for use of a financial app in their technology with all entries correct.

$(\Rightarrow \mathrm{FV} = (\pm)17389.11\ldots)$

OR

$15\,000\left(1 + \frac{3}{100}\right)^5 \left(= 17389.11\right)$ ***(A1)***

THEN

subtracting their value from their answer to part (b) ***(M1)***

$7572.17\ldots$

7570 (dollars) ***A1***

[3 marks]

9. [Maximum mark: 6]

In this question, give all answers correct to two decimal places.

Sam invests $\$1700$ in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

(a) Find the amount that Sam will have in his account after 10 years. [3]

Markscheme

Note: The first time an answer is not given to two decimal places, the final ***A1*** in that part is not awarded.

EITHER

$N = 10$	OR	$N = 20$
$I\% = 2.74$		$I\% = 2.74$
$PV = (\mp)1700$		$PV = (\mp)1700$
$P/Y = 1$		$P/Y = 2$
$C/Y = 2$		$C/Y = 2$ ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, and award ***(A1)*** for all entries correct. Accept a positive or negative value for PV.

OR

$1700\left(1 + \frac{0.0274}{2}\right)^{2\times10}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into compound interest formula.
Award ***(A1)*** for correct substitution.

THEN

$\$2231.71$ ***A1***

[3 marks]

David also invests $\$1700$ in a savings account that pays an annual rate of interest of $r\%$, compounded yearly. David makes no further payments or withdrawals from this account.

(b) Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account.

[2]

Markscheme

Note: The first time an answer is not given to two decimal places, the final ***A1*** in that part is not awarded.

EITHER

$N = 10$

$PV = \mp 1700$

$FV = \pm 2231.71\ldots$

$P/Y = 1$

$C/Y = 1$ ***(M1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen.

OR

$1700\left(1 + \frac{r}{100}\right)^{10} = 2231.71\ldots$ ***(M1)***

THEN

$r = 2.75876\ldots$

$r = 2.76$ ***A1***

Note: Ignore omission of opposite signs for PV and FV if $r = 2.76$ is obtained.

[2 marks]

(c) Find the interest David will earn over the 10 years. [1]

Markscheme

Note: The first time an answer is not given to two decimal places, the final ***A1*** in that part is not awarded.

$\$531.71$ ***A1***

[1 mark]

10. [Maximum mark: 6]

Gemma and Kaia started working for different companies on January 1st 2011.

Gemma's starting annual salary was $\$45\,000$, and her annual salary increases 2% on January 1st each year after 2011.

(a) Find Gemma's annual salary for the year 2021, to the nearest dollar. [3]

Markscheme

METHOD 1

using geometric sequence with $r = 1.02$ ***(M1)***

correct expression or listing terms correctly ***(A1)***

45000×1.02^{10} OR $45000 \times 1.02^{11-1}$ OR listing terms

Gemma's salary is $\$54855$ (must be to the nearest dollar) ***A1***

METHOD 2

$N = 10$

$\text{PV} = \mp 45000$

$\text{I\%} = 2$

$\text{P/Y} = 1$

$\text{C/Y} = 1$

$FV = \pm 54854.7489\ldots$ ***(M1)(A1)***

Gemma's salary is $\$54855$ (must be to the nearest dollar) ***A1***

[3 marks]

Kaia's annual salary is based on a yearly performance review. Her salary for the years 2011, 2013, 2014, 2018, and 2022 is shown in the following table.

year (x)	2011	2013	2014	2018	2022
annual salary ($\$S$)	45 000	47 200	48 500	53 000	57 000

(b) Assuming Kaia's annual salary can be approximately modelled by the equation $S = ax + b$, show that Kaia had a higher salary than Gemma in the year 2021, according to the model. [3]

Markscheme

finds $a = 1096.89\ldots$ and $b = -2160753.8\ldots$ (accept $b = -2.16 \times 10^6$) ***(A1)(A1)***

Note: Award ***(A1)(A1)*** for $S = 1096.89\ldots x + 33028.49\ldots$, or $S = 1096.89\ldots x + 43997.4\ldots$, or $S = 1096.89\ldots x + 45094.3\ldots$

Kaia's salary in 2021 is $\$56063.21$ (accept $\$56817.09$ from $b = -2.16 \times 10^6$) ***A1***

Kaia had a higher salary than Gemma in 2021 ***AG***

[3 marks]

11. [Maximum mark: 16]

Two friends Amelia and Bill, each set themselves a target of saving $\$20\,000$. They each have $\$9000$ to invest.

Amelia invests her $\$9000$ in an account that offers an interest rate of 7% per annum compounded **annually**.

(a.i) Find the value of Amelia's investment after 5 years to the nearest hundred dollars. [3]

Markscheme

EITHER

$9000 \times \left(1 + \frac{7}{100}\right)^5$ ***(A1)***

$12622.965\ldots$ ***(A1)***

OR

$n = 5$
$\mathrm{I}\% = 7$
$\mathrm{PV} = \mp 9000$
$\mathrm{P/Y} = 1$

$\mathrm{C/Y} = 1$ *(A1)*
$\pm 12622.965\ldots$ *(A1)*

THEN

$(\$)\,12600$ ***A1***

[3 marks]

(a.ii) Determine the number of years required for Amelia's investment to reach the target. [2]

Markscheme

EITHER

$9000\left(1 + \frac{7}{100}\right)^x = 20000$ *(A1)*

OR

$\mathrm{I\%} = 7$
$\mathrm{PV} = \mp 9000$
$\mathrm{FV} = \pm 20000$
$\mathrm{P/Y} = 1$
$\mathrm{C/Y} = 1$ *(A1)*

THEN

$= 12$ (years) ***A1***

[2 marks]

(b) Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places.

Find the minimum value of r needed for Bill to reach the target after 10 years. [3]

Markscheme

METHOD 1

attempt to substitute into compound interest formula (condone absence of compounding periods) *(M1)*

$9000\left(1+\frac{r}{100\times 12}\right)^{12\times 10}=20000$

$8.01170\ldots$ ***(A1)***

$r=8.02\,(\%)$ ***A1***

METHOD 2

$n=10$
$\mathrm{PV}=\pm 9000$
$\mathrm{FV}=\mp 20000$
$\mathrm{P/Y}=1$
$\mathrm{C/Y}=12$
$r=8.01170\ldots$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for $(\ r=)\ 8.01170\ldots$

$r=8.02\,(\%)$ ***A1***

[3 marks]

A third friend Chris also wants to reach the $\$20\,000$ target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

(c.i) Show that Chris will never reach the target if his initial deposit is $\$9000$. [5]

Markscheme

recognising geometric series (seen anywhere) ***(M1)***

$r=\frac{4500}{9000}\ \left(=\frac{1}{2}\right)$ ***(A1)***

EITHER

considering S_∞ ***(M1)***

$\frac{9000}{1-0.5}\,(=18000)$ ***A1***

correct reasoning that $18000 < 20000$ ***R1***

Note: Accept $S_\infty < 20000$ only if S_∞ has been calculated.

OR

considering S_n for a large value of $n,\ n \geq 80$ ***(M1)***

Note: Award ***M1*** only if the candidate gives a valid reason for choosing a value of n, where $50 \leq n < 80$.

correct value of S_n for their n ***A1***

valid reason why Chris will not reach the target, which involves their choice of n, their value of S_n and Chris' age OR using two large values of n to recognize asymptotic behaviour of S_n as $n \to \infty$. ***R1***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

THEN

Therefore, Chris will never reach the target. ***AG***

[5 marks]

(c.ii) Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. [3]

Markscheme

recognising geometric sum ***M1***

$\frac{u_1(1-0.5^5)}{0.5} = 20000$ ***(A1)***

$10322.58\ldots$

$(\$)\,10323$ ***A1***

[3 marks]

12. [Maximum mark: 6]

Give your answers in this question correct to the nearest whole number.

Imon invested $25\,000$ Singapore dollars (SGD) in a fixed deposit account with a nominal annual interest rate of 3.6%, compounded **monthly**.

(a) Calculate the value of Imon's investment after 5 years. [3]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure. It appeared in a paper that permitted the use of a calculator, and so might not be suitable for all forms of practice.

$(FV =)\ 25\,000 \times \left(1 + \frac{3.6}{100 \times 12}\right)^{12 \times 5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted compound interest formula, ***(A1)*** for correct substitutions.

OR

$N = 5$
$I\% = 3.6$
$PV = \mp 25\,000$
$P/Y = 1$
$C/Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 12$ seen, ***(M1)*** for **all** other correct entries.

OR

$N = 60$
$I\% = 3.6$
$PV = \mp 25\,000$
$P/Y = 12$
$C/Y = 12$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 12$ seen, ***(M1)*** for **all** other correct entries.

$(FV =)\ 29\,922\ (\text{SGD})$ ***(A1) (C3)***

Note: Do not award the final ***(A1)*** if answer is not given correct to the nearest integer.

[3 marks]

(b) At the end of the 5 years, Imon withdrew $x\ \mathrm{SGD}$ from the fixed deposit account and reinvested this into a super-savings account with a nominal annual interest rate of 5.7%, compounded **half-yearly**.

The value of the super-savings account increased to $20\,000\ \mathrm{SGD}$ after 18 months.

Find the value of x. [3]

Markscheme

$20\,000 = PV \times \left(1 + \frac{5.7}{100 \times 2}\right)^{2 \times 1.5}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted compound interest equated to $20\,000$. Award ***(A1)*** for correct substitutions.

OR

$N = 1.5$
$I\% = 5.7$
$FV = \pm 20\,000$
$P/Y = 1$
$C/Y = 2$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 2$ seen, ***(M1)*** for **all** other correct entries.

OR

$N = 3$
$I\% = 5.7$
$FV = \pm 20\,000$
$P/Y = 2$
$C/Y = 2$ ***(A1)(M1)***

Note: Award ***(A1)*** for $C/Y = 2$ seen, ***(M1)*** for **all** other correct entries.

$(x =)\ 18\,383\ (\mathrm{SGD})$ ***(A1) (C3)***

Note: Do not award the final ***(A1)*** if answer is not given correct to the nearest integer (unless already penalized in part(a)).

[3 marks]